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CONTACT TERMS, SYMMETRIES AND D-INSTANTONS.

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ABSTRACT

The scattering of $NS \otimes NS$ antisymmetric tensor states in the presence of D-instantons in type IIB superstring theory is studied. It is shown that in order to preserve gauge invariance, spacetime supersymmetry and picture changing symmetry the inclusion of boundary contact terms for closed string antisymmetric tensor vertex operators is necessary.

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1 Introduction

The gauge invariance of string scattering amplitudes manifests itself by the fact that vertex operators corresponding to longitudinal states are total derivatives on the world sheet and decouple. Such arguments can fail either when there is a contribution from the boundary of moduli space or when the world sheet itself has boundaries. One important example is given by the Cremmer-Scherk mechanism [1] where the NS \otimes NS antisymmetric tensor $B_{\mu\nu}$ and the open string vector state A_μ mix and only the combination $B_{\mu\nu} - F_{\mu\nu}$ is gauge invariant. Contact terms can be included to justify analytic continuation in external momenta (the so called ‘cancelled propagator’ argument). On the other hand unphysical states may fail to decouple in string scattering amplitudes and contact terms are needed to restore gauge invariance and unitarity [2]. In some circumstances kinematic restrictions make analytic continuation impossible and contact terms are essential [2, 3]. In this paper the scattering of closed string states is analyzed in the presence of D-instantons. In particular when NS \otimes NS antisymmetric tensors (AST) are present it is shown that the various symmetries of string perturbation theory make a boundary contact term necessary. In section 2 the BRST cohomology of the open string sector for D-instantons is analyzed. In section 3 it is shown that a NS \otimes NS AST tensor gauge transformation fails to decouple and a boundary term must be added in order to cancel this contribution. In section 4 it is shown that a space time supersymmetry transformation also induces a boundary term and is cancelled by the same contact term. In section 5 the invariance under picture changing of a closed string scattering amplitude in the one D-instanton sector is investigated and again the same contact terms are necessary for the invariance. The amplitude with two NS \otimes NS antisymmetric tensors is used as an illustration. In section 6 we present some conclusions and speculations.

2 Dirichlet instantons and BRST cohomology

The BRST charge Q_{BRST} for the open string is given by

$$Q_{BRST} = \oint dz \left\{ c \left(-\frac{1}{2} \partial X \partial X + \frac{1}{2} \psi \partial \psi - \frac{1}{2} \partial \beta \gamma - \frac{3}{2} \beta \partial \gamma \right) + c \partial c b + \frac{1}{2} \gamma \psi \partial X - \frac{1}{4} \gamma^2 b \right\}, \quad (1)$$

where b, c are the anticommuting reparameterization ghosts and β, γ are the commuting super-conformal ghosts. The space of physical vertex operators is defined as the coho-

mology class of BRST closed vertex operators $\{Q_{BRST}, V(z)\} = 0$ modulo BRST exact vertex operators $V \sim V + \{Q_{BRST}, W\}$. In the following the β, γ system is bosonized introducing a scalar ϕ which background charge 2 and a pair of weight $(1, 0)$ fermions η, ξ [4]

$$\beta = e^{-\phi} \partial \xi, \quad \gamma = \eta e^{\phi} \quad (2)$$

In terms of the bosonized fields the BRST charge is given by

$$Q_{BRST} = \oint dz \left\{ c \left(-\frac{1}{2} \partial X \partial X + \frac{1}{2} \psi \partial \psi - \frac{1}{2} (\partial \phi)^2 - 2 \partial^2 \phi \right) + c \partial c b + \frac{1}{2} e^{\phi} \eta \psi \partial X - \frac{1}{4} \eta \partial \eta b \right\}. \quad (3)$$

Note the fact that the zero mode of ξ does not enter in the bosonization formula (2), which is the starting point for the picture changing operation [4]. The β, γ system is a commuting first order system which has infinitely many distinct vacua (pictures) which are labelled by the superghost charge $j = \oint \partial \phi$.

The picture changing operator \mathcal{X} which maps a vertex operator in a picture s to a vertex operator in picture $s + 1$, $\mathcal{X} : V_s \rightarrow V_{s+1}$ by

$$V_{s+1}(z) = \{Q_{BRST}, \xi V_s(z)\}. \quad (4)$$

In the standard Neumann open string theory the (fixed) vertex operator for the massless vector state is given by in the $s = -1$ and $s = 0$ picture respectively

$$V_{-1} = c e^{-\phi} \psi^{\mu} e^{ikX}, \quad V_0 = c (\partial X^{\mu} + i k_{\rho} \psi^{\rho} \psi^{\mu}) e^{ikX}. \quad (5)$$

It is easy to see that the two vertex operators are indeed related by (4) and that the vertex operator represent physical states in an appropriate gauge if $k_{\mu} \zeta^{\mu} = 0$ and $k^2 = 0$. In the Neveu-Schwarz sector there is a simpler and less sophisticated description of the picture changing. One can define two pictures called F_1 and F_2 [16]. For open strings the NS-sector states F_2 picture obeys the mass shell condition $(L_0 - \frac{1}{2}) | \phi \rangle = 0$ and $G_r | \phi \rangle = L_n | \phi \rangle = 0$ for $r > 0, n > 0$. Because $G_{1/2} G_{-1/2} | \phi \rangle_{F_2} = | \phi \rangle_{F_2}$ one can define an open string state in the F_1 picture defined by $| \phi \rangle_{F_1} = G_{-1/2} | \phi \rangle_{F_2}$. For the open string with Neumann boundary conditions the massless vector states take the following form in the two pictures.

$$| \zeta_{\mu}, k \rangle_{F_2} = \zeta_{\mu} \psi_{-1/2}^{\mu} | k \rangle, \quad (6)$$

$$| \zeta_{\mu}, k \rangle_{F_1} = \zeta_{\mu} (a_{-1}^{\mu} + i k_{\rho} \psi_{-1/2}^{\rho} \psi_{-1/2}^{\mu}) | k \rangle. \quad (7)$$

Which shows that the state (7) in the F_1 picture is given by a worldsheet supersymmetry transformation of the state (6) in the F_2 picture.

D-instanton boundary conditions on the open string coordinate X^μ are defined by imposing $X^\mu|_{\sigma=0} = Y_1^\mu$ and $X^\mu|_{\sigma=\pi} = Y_2^\mu$. The mode expansion is then given by

$$X^\mu = Y_1^\mu + \frac{Y_2^\mu - Y_1^\mu}{\pi}\sigma + \sqrt{2\alpha'} \sum_n \frac{a_n^\mu}{n} e^{-in\tau} \sin(n\sigma). \quad (8)$$

The zero mode of the matter energy momentum tensor for an open string ‘stretched’ between two D-instantons at Y_1^μ and Y_2^μ is given by

$$L_0^X = \frac{(Y_1 - Y_2)^2}{4\pi\alpha'} + \sum_n : a_{-n}^\mu a_{-n\mu} : . \quad (9)$$

The usual open string momentum for an open string satisfying Neumann boundary conditions is replaced by the separation $(Y_1^\mu - Y_2^\mu)/\alpha'$ for the D-instanton.

For a single D-instanton which is located at y^μ we have $Y_1^\mu = Y_2^\mu = y^\mu$. This implies that the BRST cohomology for D-instanton open string states is isomorphic to the cohomology of Neumann open strings at zero momentum [5]. The open string states do not carry any momentum and the vertex operator for the level one vector state is given by

$$V_{-1} = \zeta_\mu \int dx e^{-\phi} \psi^\mu, \quad V_0 = \zeta_\mu \int dx \partial_n X^\mu. \quad (10)$$

It is easy to see that both vertex operators commute with the BRST charge (3) without imposing any constraint on the wavefunction ζ_μ which is interpreted as a collective coordinate corresponding to the shift in the position of the D-instanton y^μ in space time. Polchinski has shown that upon integration over the instanton position y^μ all divergences associated with boundary of moduli space cancel [6]. There is another level one state which appears in the vertex V_0 (5)

$$V_0^c = \zeta_{\mu\nu} \oint dx \psi^\mu \psi^\nu e^{iky}. \quad (11)$$

In the case of the D-instanton this is not BRST invariant for nonvanishing $\zeta_{\mu\nu}$ since $\{Q_{BRST}, V_0^c\} = \zeta_{\mu\nu} \oint dx e^\phi \eta \psi^\mu \partial X^\nu$ and therefore unphysical. As we shall see in the subsequent section this unphysical state will become important as a contact term which restores closed string symmetries in the presence of D-instantons.

3 Gauge invariance

The effect of boundaries with D-instanton boundary conditions is studied in the simplest case where the worldsheet is the upper half plane $H = \{z | \text{Im}(z) > 0\}$. Dirichlet boundary conditions are imposed on all the coordinates and the boundary conditions on the worldsheet fermions are determined by worldsheet supersymmetry [7].

$$\left(\partial X^\mu(z) + \bar{\partial} X^\mu(\bar{z})\right) |_{\text{Im}(z)=0} = 0, \quad \left(\psi^\mu(z) + \bar{\psi}^\mu(\bar{z})\right) |_{\text{Im}(z)=0} = 0. \quad (12)$$

The boundary is given by the real line, open string vertex operators are written in terms of leftmoving oscillators only and correlation functions can be evaluated by the well known doubling procedure [8]. The vertex operator for the $\text{NS} \otimes \text{NS}$ state in the $(0,0)$ picture is given by

$$V(\zeta) = \zeta_{\mu\nu} (\partial X^\mu + ik_\rho \psi^\rho \psi^\mu) (\bar{\partial} X^\nu + ik_\lambda \bar{\psi}^\lambda \bar{\psi}^\nu) e^{ikX}(z, \bar{z}). \quad (13)$$

The physical state conditions for the massless tensor states imply that $\zeta_{\mu\nu} k^\mu = \zeta_{\mu\nu} k^\nu = 0$ and $k^2 = 0$. The tensor $\zeta_{\mu\nu}$ decomposes into the traceless symmetric part (graviton), dilaton and antisymmetric part. For the graviton and antisymmetric tensor the gauge invariance associated manifests itself in the fact that upon the replacement $\delta\zeta^{(1)} = k_\mu \Lambda_\nu$ and $\delta\zeta_{\mu\nu}^{(2)} = k_\nu \Lambda_\mu$ the vertex operator (13) becomes a total derivative which decouples on a compact worldsheet.

$$V(\delta\zeta^{(1)}) = \Lambda_\nu \partial \left\{ (\bar{\partial} X^\nu + ik_\lambda \bar{\psi}^\lambda \bar{\psi}^\nu) e^{ikX} \right\}, \quad (14)$$

$$V(\delta\zeta^{(2)}) = \Lambda_\nu \bar{\partial} \left\{ (\partial X^\nu + ik_\lambda \psi^\lambda \psi^\nu) e^{ikX} \right\}. \quad (15)$$

A gauge transformation on the graviton wavefunction corresponds to a linearized coordinate transformation and is given by $\delta\zeta_{\mu\nu}^{grav} = \delta\zeta_{\mu\nu}^{(1)} + \delta\zeta_{\mu\nu}^{(2)}$. For D-instanton boundary conditions the total derivatives in (14) and (15) produce a boundary term of the form

$$\int d^2z V(\delta\zeta^{grav}) = \Lambda_\nu \int dx \partial_n X^\nu = \Lambda_\nu \frac{\partial}{\partial Y_\nu}. \quad (16)$$

The boundary operator is proportional to the momentum conjugate to the position of the D-instanton and the gauge transformation induces an infinitesimal shift in the position of the D-instanton $\delta Y_\nu = \Lambda_\nu$.

For the antisymmetric tensor a gauge transformation is given by $\delta\zeta_{\mu\nu}^{AST} = \delta\zeta_{\mu\nu}^{(1)} - \delta\zeta_{\mu\nu}^{(2)}$ and yield the following boundary term

$$\int d^2z V(\delta\zeta^{AST}) = i\Lambda_{[\nu} k_{\mu]} \int dx \psi^\mu \psi^\nu e^{iky}. \quad (17)$$

In order to preserve gauge invariance a boundary term (11) has to be added where the wavefunction $\zeta_{\mu\nu}$ transforms $\delta\zeta_{\mu\nu} = k_{[\mu}\Lambda_{\nu]}$ under gauge transformations and therefore cancels a boundary term (17).

The boundary conditions (12) reflect left and rightmoving degrees of freedom. This leads to nontrivial OPE between left and rightmovers for closed string vertex operators coming close to the boundary which can be used to deduce the coupling of closed and open strings [9]. For a NS \otimes NS AST vertex operator located at $z = x + iy$ the singular part of the boundary induced OPE is given by

$$\begin{aligned} \lim_{y \rightarrow 0} V(\zeta_{\mu\nu}^{AST}) &\sim \zeta_{\mu\nu}^{AST} \frac{1}{(y)^{k^2+1}} k^2 \psi^\mu \psi^\nu(x) + o(y^0) \\ &= - \left(\frac{\partial}{\partial y} y^{k^2} \right) \zeta_{\mu\nu}^{AST} \psi^\mu \psi^\nu(x) + o(y^0). \end{aligned} \quad (18)$$

This situation is very similar to the one discussed in [2], for the integrated vertex operator the total derivative gives $\int dy \frac{\partial}{\partial y} y^{k^2} = y^{k^2} |_{y=0}$. For $k^2 > 0$ this term vanishes whereas for $k^2 < 0$ it is divergent but the on-shell condition for the closed string vertex operator enforces $k^2 = 0$. The contact term at the boundary is therefore necessary to cancel the total derivative and to restore the gauge invariance.

Note that the contact term (11) can be absorbed into the the AST vertex operator in order to write it partly in a manifestly gauge invariant fashion. Using the boundary conditions (12) the boundary contact term is reexpressed as a bulk vertex,

$$\begin{aligned} \zeta_{\mu\nu} \int dx \psi^\mu \psi^\nu e^{iky} &= \frac{1}{2} \zeta_{\mu\nu} \int d^2 z \partial \left(\bar{\psi}^\mu \bar{\psi}^\nu e^{ikX} \right) - \frac{1}{2} \zeta_{\mu\nu} \int d^2 z \bar{\partial} \left(\psi^\mu \psi^\nu e^{ikX} \right) \\ &= i \frac{1}{2} \zeta_{\mu\nu} k_\rho \int d^2 z \left(\psi^\mu \psi^\nu \bar{\partial} X^\rho - \bar{\psi}^\mu \bar{\psi}^\nu \partial X^\rho \right) e^{ikX}. \end{aligned} \quad (19)$$

In this form it can be added to the part of the AST vertex (13) which fails to decouple under linearized gauge transformations namely the terms

$$\begin{aligned} &i \zeta_{\mu\nu} k_\rho \int d^2 z \left(\partial X^\mu \bar{\psi}^\rho \bar{\psi}^\nu - \bar{\partial} X^\mu \psi^\rho \psi^\nu \right) e^{ikX} + \zeta_{\mu\nu} \int dx \psi^\mu \psi^\nu e^{iky} \\ &= \frac{1}{2} H_{\mu\nu\rho} \int d^2 z \left(\partial X^\mu \bar{\psi}^\rho \bar{\psi}^\nu - \bar{\partial} X^\mu \psi^\rho \psi^\nu \right) e^{ikX}, \end{aligned} \quad (20)$$

where (19) was used and $H_{\mu\nu\rho} = i \frac{1}{2} \zeta_{[\mu\nu} k_{\rho]}$ is the manifestly gauge invariant field strength for the two form AST potential $\zeta_{\mu\nu}$.

4 Space time supersymmetry

In the Neveu-Schwarz-Ramond formalism the space time supersymmetry generators are constructed from spin fields and superghosts [4]. The charges in the $s = -1/2$ and $s = 1/2$ picture are given by

$$Q_{-1/2}^a = \oint dz e^{-1/2\phi} S^a, \quad Q_{1/2}^a = \oint dz e^{1/2\phi} (\gamma^\mu)_b^a S^b \partial X_\mu. \quad (21)$$

Similarly for the rightmoving supercharges \bar{Q}_s^a . Because of the superghost dependent part in (21) the spacetime supersymmetry charges appear in different pictures. Note that the supersymmetry algebra only closes on shell, i.e. modulo picture changing operation. The anticommutator $\{Q_{-1/2}^a, Q_{-1/2}^b\} = \gamma_\mu^{ab} \oint dz e^{-\phi} \psi^\mu$ is related by picture changing to $\{Q_{1/2}^a, Q_{-1/2}^b\} = \gamma_\mu^{ab} \oint dz \partial X^\mu$ where the standard space time momentum operator appears on the right hand side. The supersymmetry transformation on the vertex operators (13) is only closing up to total derivatives which again can yield boundary terms. Indeed using standard OPE techniques one can show that the anti-commutator of the the supersymmetry charge in the $s = 1/2$ picture and the leftmoving part (13) is given by

$$\begin{aligned} \{Q_{1/2}^a, \zeta_\mu (\partial X^\mu + ik_\rho \psi^\rho \psi^\mu) e^{ikX}(z)\} &= \zeta_\mu \oint dw \frac{1}{(z-w)^2} e^{1/2\phi} S^b(w) (\gamma^\mu)_b^a e^{ikX}(z) \\ &- ik_\nu \zeta_\mu \oint dw \frac{1}{z-w} e^{1/2\phi} S^b(w) (\gamma^\nu)_b^a \partial X^\mu e^{ikX}(z) \\ &+ ik_\nu ik_\rho \zeta_\mu \oint dw \frac{1}{(z-w)^2} e^{1/2\phi} S^b(w) (\gamma^\nu \gamma^{\rho\mu})_b^a e^{ikX}(z) \\ &+ ik_\rho \zeta_\mu \oint dw \frac{1}{z-w} e^{1/2\phi} S^b \partial X_\nu(w) (\gamma^\nu \gamma^{\rho\mu})_b^a e^{ikX}(z). \end{aligned} \quad (22)$$

Note that the third term on the right hand side of (22) vanishes because of the the two momentum factors and the physical state condition on the polarization vector ζ_μ . Rearranging the gamma matrices using $(\gamma^\nu \gamma^{\rho\mu})_b^a = \delta^{\nu\rho} (\gamma^\mu)_b^a - \delta^{\nu\mu} (\gamma^\rho)_b^a + (\gamma^{\rho\mu} \gamma^\nu)_b^a$ the other terms can be combined in the following form

$$\begin{aligned} \{Q_{1/2}^a, \zeta_\mu (\partial X^\mu + ik_\rho \psi^\rho \psi^\mu) e^{ikX}(z)\} &= -\partial \left(\zeta_\mu e^{\phi/2} S^b (\gamma^\mu)_b^a e^{ikX} \right) \\ &- ik_\rho \zeta_\mu e^{\phi/2} S^b \partial X_\nu (\gamma^{\rho\mu} \gamma^\nu)_b^a e^{ikX}. \end{aligned} \quad (23)$$

Similarly the anticommutator of rightmoving supercharge and the rightmoving part of the vertex (13) is given by

$$\begin{aligned} \{\bar{Q}_{1/2}^a, \bar{\zeta}_\mu (\bar{\partial} X^\mu + ik_\rho \bar{\psi}^\rho \bar{\psi}^\mu) e^{ikX}(\bar{z})\} &= -\bar{\partial} (\bar{\zeta}_\mu e^{\bar{\phi}/2} \bar{S}^b (\gamma^\mu)_b^a e^{ikX}) \\ &- ik_\rho \bar{\zeta}_\mu e^{\bar{\phi}/2} \bar{S}^b \bar{\partial} X_\nu (\gamma^{\rho\mu} \gamma^\nu)_b^a e^{ikX}. \end{aligned} \quad (24)$$

The supersymmetry transformation of the vertices produces a total derivative and a term which corresponds to the linearized supersymmetry transformation on the spinor in standard open string supersymmetric Yang-Mills $\delta\psi = \Gamma^{\mu\nu} F_{\mu\nu} \epsilon$. The supersymmetries in the closed string theory are generated by both left and right moving supercharges. The boundary conditions on the fields X^μ, ψ^μ (12) together with the consistency of the operator product expansion imply the following boundary condition² on the spin fields [10, 11].

$$(S^a(z) + i(\gamma^{11})_b^a \bar{S}^b(\bar{z}))|_{Im(z)=0} = 0. \quad (25)$$

Defining the two combinations of supercharges

$$Q_s^{a+} = Q_s^a + i(\gamma^{11})_b^a \bar{Q}_s^b, \quad Q_s^{a-} = Q_s^a - (\gamma^{11})_b^a i\bar{Q}_s^b, \quad (26)$$

it is easy to see that the boundary conditions imply that Q^{a+} vanishes at the boundary and represents the unbroken supersymmetry. Whereas Q^{a-} does not vanish and corresponds to the broken supersymmetry. It has the form of a zero momentum open string fermionic vertex operator and represents the fermionic zero mode produced by the broken supersymmetry [12, 13].

We consider the effect of an unbroken supersymmetry transformation on the NS \otimes NS antisymmetric tensor (13). Combining (23) and (24) gives

$$\{\epsilon_a^+ (Q_{1/2}^a + i\bar{Q}_{1/2}^a), \int d^2z V_{(0,0)}(\zeta_{\mu\nu}^{AST})\} = \int d^2z \delta_{\epsilon^+} V + \epsilon_a^+ \zeta_{\mu\nu}^{AST} \int dx e^{\phi/2} S^b \partial X^{[\mu} (\gamma^{\nu]}_b^a, \quad (27)$$

where $\delta_{\epsilon^+} V$ is the standard linearized supersymmetry transformation of the NS \otimes NS antisymmetric tensor. In deriving the boundary contribution one has to carefully take into account the boundary conditions (12),(25) in order to write all the fields on the boundary in terms of leftmovers. Hence the supersymmetry transformation of the NS \otimes NS AST produces a nonvanishing boundary term which is cancelled by the action of the

²The factor of i appears because analytic continuation to euclidean signature for the D-instanton.

supercharge on the boundary term (11).

$$\begin{aligned}
\{\epsilon_a^+(\{Q_{1/2}^a + i\bar{Q}_{1/2}^a\}), \zeta_{\mu\nu} \int dx \psi^\mu \psi^\nu(x)\} &= \epsilon_a^+ \oint dw e^{1/2\phi} (\gamma^\rho)_b^a \partial X_\rho \zeta_{\mu\nu} \oint dx \psi^\mu \psi^\nu \\
&= \epsilon_a^+ \zeta_{\mu\nu} \oint dx \oint dw \frac{1}{w-x} e^{1/2\phi} S^c (\gamma^\rho \gamma^{\mu\nu})_c^a \\
&= \epsilon_a^+ \zeta_{\mu\nu} \int dx e^{1/2\phi} (\gamma^{[\mu})_b^a \partial X^{\nu]} S^b + \zeta_{\mu\nu} \epsilon_a^+ (\gamma^{\mu\nu})_b^a Q_{1/2}^b.
\end{aligned} \tag{28}$$

The first term in (28) exactly cancels the boundary term in (27). The second term has the form of a shift in the fermionic collective coordinate. Here are two comments in order. Firstly the total derivative in (27) appears because the supercharge $Q_{1/2}^a$ in the $s = 1/2$ picture was used. It is easy to see that the same calculation using the supercharge in the $s = -1/2$ picture $Q_{-1/2}^{a+}$ does not lead to a total derivative term in $\{Q_{-1/2}^{a+}, V_{(0,0)}(\zeta_{\mu\nu})\}$. This is consistent with the fact that the commutator of the supercharge in the $s = -1/2$ picture with the boundary term (11) only produces the shift in the fermionic collective coordinate,

$$\{\epsilon_a^+(\{Q_{-1/2}^a + i\bar{Q}_{-1/2}^a\}), \int dx \zeta_{\mu\nu} \psi^\mu \psi^\nu(x)\} = \zeta_{\mu\nu} \epsilon_a^+ (\gamma^{\mu\nu})_b^a Q_{-1/2}^b. \tag{29}$$

Secondly, the fact that the broken supersymmetry is nonlinearly realized on the open string degrees of freedom and acts as a shift in the fermionic collective coordinate has recently played an important part in the IIB matrix model [14]. The supersymmetry algebra for the zero dimensional reduction of $U(n)$ SYM theory to zero dimensions

$$\delta A_\mu = \bar{\epsilon} \Gamma_\mu \psi, \quad \delta \psi = i[A_\mu, A_\nu] \Gamma^{\mu\nu} \epsilon + \eta. \tag{30}$$

Here ϵ and η corresponds to the unbroken and broken supersymmetry respectively. Here we suppressed the $U(n)$ Lie algebra indices but note that the shift $\delta\psi = \eta$ lies in the $U(1)$ part of $U(n) = SU(n) \times U(1)$ which corresponds to the overall 'center of mass' degrees of freedom. The nonabelian structure $\delta\psi$ becomes manifest when Chan–Paton factors and open string contact terms are considered [15]. The fact that the supersymmetry is changed by the presence of boundary terms seems to indicate that the closed string AST background influences the nonlinearly realized open string supersymmetry.

5 Picture changing

In the presence of closed string $\text{NS} \otimes \text{NS}$ antisymmetric tensor states the independence of on-shell scattering amplitudes from the choice of picture makes the presence of boundary contact terms (11) necessary. In this section the half plane is conformally mapped into the semi-infinite cylinder and the D-instanton boundary conditions are encoded in a Boundary state $|B\rangle$ [10, 11, 12]. Using the operator formalism the n -point amplitude for n $\text{NS} \otimes \text{NS}$ massless tensor states is given in the (F_2, F_2) -picture

$$A_n =_{F_2, F_2} \langle \zeta_1, k_1 | V(\zeta_2, k_2) \Delta \cdots \Delta V(\zeta_n, k_n) \Delta | B \rangle. \quad (31)$$

Where the state in F_2, F_2 picture is defined by

$$_{F_2, F_2} \langle \zeta_1, k_1 | = \zeta_{\mu\nu}^{(1)} \langle k_1 | b_{1/2}^\mu \bar{b}_{1/2}^\nu. \quad (32)$$

The closed string propagator is given by

$$\Delta = P_{L_0 - \bar{L}_0} \frac{1}{L_0 + \bar{L}_0 - 1} \quad , \quad P_{L_0 - \bar{L}_0} = \frac{1}{2\pi} \int_0^{2\pi} d\sigma e^{i\sigma(L_0 - \bar{L}_0)}. \quad (33)$$

To prove the independence of the amplitude of the picture chosen, we turn the state (32) into a state in the picture (F_1, F_2)

$$_{F_2, F_2} \langle \zeta_1, k_1 | =_{F_1, F_2} \langle \zeta_1, k_1 | G_{-1/2}. \quad (34)$$

We want to show that A_n in the (F_2, F_2) is equivalent to A_n in the (F_1, F_1) picture. Inserting (34) into (31), $G_{-1/2}$ is then moved to the right until it hits the boundary state $|B\rangle$ where it is reflected into a $i\bar{G}_{1/2}$ it is then moved to the left, where it turns the (F_1, F_2) into a (F_1, F_1) state by

$$_{F_1, F_2} \langle \zeta_1, k_1 | \bar{G}_{1/2} =_{F_1, F_1} \langle \zeta_1, k_1 |. \quad (35)$$

Moving the $G_{1/2}$ to the right has two effects: firstly using $[L_0, G_r] = -rG_r$ the propagator gets shifted by

$$G_{-1/2} \Delta = \Delta' G_{-1/2} \quad , \quad \Delta' = P_{L_0 - \bar{L}_0 - 1/2} \frac{1}{L_0 + \bar{L}_0 - 3/2}. \quad (36)$$

Secondly we have to show that the commutator $[G_{-1/2}, V(m)]$ vanishes for $m = 2, \dots, n$. Note that the vertex operators are products of left and right-moving pieces, i.e. $V =$

$\zeta_{\mu\nu} V_L^\mu V_R^\nu$. It is well known that for the physical vertex operator V there exists a $W = \zeta_{\mu\nu} W_L^\mu V_R^\nu$ and $W' = \zeta_{\mu\nu} V_L^\mu W_R^\nu$ such that $[G_{-1/2}, V] = [L_{-1}, W]$ and $[\bar{G}_{1/2}, V] = [\bar{L}_1, W']$, (see for example [16]). The left and right-moving parts of W are operators of left and right-moving conformal dimension $1/2$ respectively. Hence the commutator can be replaced by

$$\begin{aligned} [G_{-1/2}, V] &\rightarrow (L_{-1} - L_0 + 1)W - W(L_{-1} - L_0 + 1/2) \\ &+ (L_0 - 1)W - W(L_0 - 1/2). \end{aligned} \quad (37)$$

Where we added and subtracted the second line. The first line vanishes because the left-moving part W_L^μ has conformal dimension of $J = 1/2$. The second line does not contribute to the on-shell scattering amplitude because of the cancelled propagator argument. Using the level matching projectors the propagators Δ and Δ' can be written as purely left-moving operators

$$\Delta = \frac{1}{2(L_0 - 1/2)} \quad , \quad \Delta' = \frac{1}{2(L_0 - 1)}. \quad (38)$$

Where in the second line of (37) $L_0 - 1$ cancels the shifted propagator Δ' to the left and $L_0 - 1/2$ cancels Δ to the right. When the $G_{-1/2}$ hits the boundary state it is reflected according to

$$G_{-1/2} | B \rangle = i \bar{G}_{1/2} | B \rangle. \quad (39)$$

The reflected $\bar{G}_{1/2}$ is moved to the left where again shifts propagators according to

$$\Delta' \bar{G}_{1/2} = \bar{G}_{1/2} \Delta'' \quad , \quad \Delta'' = P_{L_0 - \bar{L}_0} \frac{1}{L_0 + \bar{L}_0 - 2}. \quad (40)$$

The commutator $[\bar{G}_{1/2}, V] = [\bar{L}_1, W']$ can be replaced by

$$\begin{aligned} [\bar{G}_{1/2}, V] &= (\bar{L}_1 - \bar{L}_0)W' - W'(\bar{L}_1 - \bar{L}_0 + 1/2) \\ &+ (\bar{L}_0 - \frac{1}{2})W' - W'(\bar{L}_0 - 1). \end{aligned} \quad (41)$$

Where again we added and subtracted zero. The first line vanishes now because the right-moving part of W' has $\bar{J} = 1/2$. The second line cancels the propagators to the left and right

$$\Delta' = \frac{1}{2(\bar{L}_0 - 1/2)} \quad , \quad \Delta'' = \frac{1}{2(\bar{L}_0 - 1)}. \quad (42)$$

Note that Δ' is different form (38) because the propagators are now expressed in right-moving oscillators. When the $\bar{G}_{1/2}$ hits the first state it changes its picture (35) and The

amplitude A_n is equal to the picture changed version where ${}_{F_2 F_2} \langle 1 |$ is replaced by ${}_{F_1 F_1} \langle 1 |$ and Δ by Δ'' . This argument fails to be valid because the cancelled propagator for the vertex closest to the boundary produces a contact term which was not considered.

$$\cdots \left([G_{-1/2}, V_n] \Delta - i[\bar{G}_{1/2}, V_n] \Delta'' \right) | B \rangle \quad (43)$$

The explicit form of the vertices for the massless states is given by $V^\mu = (\partial X^\mu + ik_\rho \psi^\rho \psi^\mu) e^{ikX}$ and $W^\mu = \psi^\mu e^{ikX}$. Using (37) and (41) the explicit form of the contact term for massless NS \otimes NS tensor states is given by

$$\frac{1}{2} \zeta_{\mu\nu}^{(n)} \sum_m \left(-b_{m-1/2}^\mu (\bar{a}_m^\nu + ik^\rho \sum_s \bar{b}_{m-s}^\rho \bar{b}_s^\nu) + i\bar{b}_{m+1/2}^\nu (a_m^\mu + ik^\rho \sum_s b_{m-s}^\rho b_s^\mu) \right) | B \rangle. \quad (44)$$

The boundary conditions on the modes can be used to write the contact term entirely in terms of left-moving oscillators. It follows that the terms trilinear in the fermionic modes vanish and that the contact term is given by

$$\frac{1}{2} \zeta_{\mu\nu}^{(n)} \sum_m \left(-a_m^\nu b_{-m-1/2}^\mu + a_{-m}^\mu b_{m-1/2}^\nu \right) | B \rangle. \quad (45)$$

Note that only an antisymmetric part of $\zeta_{\mu\nu}^{(n)}$ contributes for the boundary term. This contact term can be cancelled by an additional boundary term in A_n given by

$$A_n \rightarrow A_n + {}_{F_2, F_2} \langle 1 | V(2) \Delta \cdots \Delta V(n-1) \Delta K(n) | B \rangle. \quad (46)$$

Where the boundary operator is given by

$$K(n) = -i \zeta_{\mu\nu}^{(n)} \sum_{r>0} b_{-r}^\mu \bar{b}_{-r}^\nu. \quad (47)$$

The additional term produces a commutator in the picture changing procedure

$$\begin{aligned} [G_{-1/2} - i\bar{G}_{1/2}, K(n)] | B \rangle &= i \zeta_{\mu\nu}^{(n)} \sum_{m<1/2} \left(a_m^\mu \bar{b}_{m-1/2}^\nu + i\bar{a}_m^\nu b_{m+1/2}^\mu \right) | B \rangle \\ &= \frac{1}{2} \zeta_{\mu\nu}^{(n)} \sum_{m=-\infty}^{\infty} \left(-a_m^\mu b_{-m+1/2}^\nu + a_m^\nu b_{-m+1/2}^\mu \right). \end{aligned} \quad (48)$$

The second line is written in terms of left-moving oscillators and the factor $1/2$ is introduced in order to double the range of summation. It is now easy to see that the boundary

term (48) cancels (45). This is not the full story since the cancelled propagator argument for the extra term now produces a boundary contribution for the vertex V_{n-1}

$$\cdots \left([G_{-1/2}, V(n-1)]\Delta - i[\bar{G}_{1/2}, V(n-1)]\Delta'' \right) K(n) | B \rangle, \quad (49)$$

which can be cancelled by adding another boundary term $\langle 1 | V(1) \cdots V(n-2) K(n-1) K(n) | B \rangle$ and so on.

It is easy to see that all the contact terms are cancelled in the following sum

$$\begin{aligned} A'_n &= {}_{F_2, F_2} \langle 1 | V(2) \Delta \cdots \Delta V(n) \Delta | B \rangle \\ &+ {}_{F_2, F_2} \langle 1 | V(2) \Delta \cdots \Delta V(n-1) \Delta K(n) | B \rangle \\ &+ {}_{F_2, F_2} \langle 1 | V(2) \Delta \cdots \Delta V(n-2) \Delta K(n-1) K(n) | B \rangle \\ &\vdots \\ &+ {}_{F_2, F_2} \langle 1 | \Delta K(2) \cdots K(n) | B \rangle + \text{other orderings}. \end{aligned} \quad (50)$$

Where ‘other orderings’ denotes the different time orderings on the cylinder which have to be added. Note that the K ’s commute since they are made out of creation operators. The corrected amplitude A'_n is now invariant under the picture changing operation and A_n is equal to (50) with ${}_{F_2, F_2} \langle 1 |$ replaced by ${}_{F_1, F_1} \langle 1 |$ and Δ replaced by Δ''

The appearance of the boundary terms can be understood in the cylinder frame by considering the inclusion of a boundary term in the action which is represented as a operator acting on the boundary state

$$Z = \langle 0 | \exp(iS_{bulk}) \exp(iB_{\mu\nu} \oint \psi^\mu \bar{\psi}^\nu) | B \rangle. \quad (51)$$

The scattering amplitude is given by

$$A_n = \prod_{k=1}^n \left(\zeta_{k, \mu_k} \bar{\zeta}_{k, \nu_k} \frac{\delta}{\delta B_{\mu_k \nu_k}} \right) Z, \quad (52)$$

which produces all the terms given in (50).

To illustrate the necessity of the contact terms we will consider amplitude for two $NS \otimes NS$ antisymmetric tensors on the disk with D-instanton boundary conditions. This amplitude was first calculated by Klebanov and Thorlacius in [17].

$$A_2 = \int dy \left\langle c \bar{c} V_{-2,0}(\zeta^{(1)}, k_1) (c + \bar{c}) V_{0,0}(\zeta^{(2)}, k_2) (iy) \right\rangle = \frac{1}{k_1 \cdot k_2} H_{\mu\nu\rho}^{(1)} H^{(2)\mu\nu\rho}. \quad (53)$$

Where $H_{\mu\nu\rho}^{(i)} = ik_{[\mu}^{(i)}\zeta_{\nu\rho]}^{(i)}$ for $i = 1, 2$. Note that the important part of the vertex $V_{-2,0}$ in (53) is up to a super-ghost factor of the same form as $V_{0,0}$ defined in (13). We focus on the contribution to (53) is given by $1/2(\zeta_{\mu\nu}^{(1)}\zeta^{(2)\mu\nu} - \zeta_{\nu\mu}^{(1)}\zeta^{(2)\mu\nu})$. The two point function can be calculated in a different picture defined by

$$A'_2 = \int dy \langle c\bar{c}V_{-1,-1}(\zeta^{(1)}, k_1)(c + \bar{c})V_{0,0}(\zeta^{(2)}, k_2)(iy) \rangle. \quad (54)$$

Where the vertex operator in the (-1,-1) picture is given by

$$V_{-1,-1}(\zeta) = \zeta_{\mu\nu}e^{-\phi}e^{-\bar{\phi}}\psi^\mu\bar{\psi}^\nu \quad (55)$$

Without the boundary term the contractions which produce $\zeta_{\mu\nu}^{(1)}\zeta^{(2)\mu\nu} - \zeta_{\nu\mu}^{(1)}\zeta^{(2)\mu\nu}$ are given by

$$\begin{aligned} A'_2 &= \zeta_{\mu\nu}^{(1)}\zeta^{(2)\mu\nu} \int dy(1-y)^{2k_1.k_2-1}(1+y)^{2k_1.k_2+1}y^{-1-k_2^2}k_2^2 \\ &\quad - \zeta_{\nu\mu}^{(1)}\zeta^{(2)\mu\nu} \int dy(1-y)^{2k_1.k_2+1}(1+y)^{2k_1.k_2-1}y^{-1-k_2^2}k_2^2 + \text{other terms}. \end{aligned} \quad (56)$$

Since these terms are proportional to k_2^2 they vanish because of the on-shell condition for the massless antisymmetric tensor state $k_2^2 = 0$. However if one allows the state to be slightly off shell, we can use the following formula

$$\int dy(1-y)^a(1+y)^by^c = 2^{-(2c+2)} \frac{\Gamma(\frac{a+1}{2})\Gamma(c+1)}{\Gamma(\frac{a+1}{2} + c + 1)}, \quad (57)$$

which is valid for $a + b + 2c + 2 = 0$ to find that the integral (56) is given by

$$\zeta_{\mu\nu}^{(1)}\zeta^{(2)\mu\nu}k_2^2 \frac{\Gamma(k_1.k_2)\Gamma(k_2^2)}{\Gamma(k_1.k_2 + k_2^2)} - \zeta_{\nu\mu}^{(1)}\zeta^{(2)\mu\nu}k_2^2 \frac{\Gamma(k_1.k_2 + 1)\Gamma(k_2^2)}{\Gamma(k_1.k_2 + 1 + k_2^2)}. \quad (58)$$

The factor of k_2^2 is cancelled by a pole coming from the Γ function, i.e. $k_2^2\Gamma(k_2^2) = \Gamma(k_2^2 + 1) \rightarrow 1$ as $k_2^2 \rightarrow 0$. Hence if we allow the vertex operators to be slightly off-shell the amplitude (56) contains the term of the form $\zeta_{\mu\nu}^{(1)}\zeta^{(2)\mu\nu} - \zeta_{\nu\mu}^{(1)}\zeta^{(2)\mu\nu}$. But conformal invariance requires the vertex operators to be on shell and this continuation can only be regarded to be a trick to get the correct gauge invariant result. Indeed the amplitudes written as functional integrals on the half plane can be represented in the cylinder frame and the two point functions in the two different pictures correspond to

$$A_2 = {}_{F_1F_1} \langle \zeta^{(1)} | V(\zeta^{(2)})\Delta | B \rangle \quad (59)$$

$$A'_2 = {}_{F_2F_2} \langle \zeta^{(1)} | V(\zeta^{(2)})\Delta | B \rangle. \quad (60)$$

It was argued above that it is necessary to introduce a boundary term when $\text{NS} \otimes \text{NS}$ antisymmetric tensor vertex operators are present. For the amplitude A'_2 such a boundary term is given by

$$\begin{aligned} {}_{F_2 F_2} \langle \zeta_{\mu\nu}^{(1)} | K(\zeta^{(2)}) | B \rangle &= \zeta_{\mu\nu}^{(1)} \zeta_{\rho\lambda}^{(2)} \langle k_1 | b_{1/2}^\mu \bar{b}_{1/2}^\nu (b_{1/2}^\lambda \bar{b}_{1/2}^\rho - b_{1/2}^\rho \bar{b}_{1/2}^\lambda) | B \rangle \\ &= \zeta_{\mu\nu}^{(1)} \zeta^{(2)\mu\nu} - \zeta_{\nu\mu}^{(1)} \zeta^{(2)\mu\nu}. \end{aligned} \quad (61)$$

The inclusion of the boundary term completes the gauge invariant result (53) which was calculated in a different picture hence showing that the calculation of the two point function for $\text{NS} \otimes \text{NS}$ antisymmetric tensor states in the two pictures are equivalent once the boundary terms are included.

6 Conclusions

In this paper it was shown that the presence of D-instantons makes the inclusion of boundary contact terms for $\text{NS} \otimes \text{NS}$ antisymmetric tensor fields necessary. This situation is analogous to the one discussed in [2]. We have seen that the contact terms are necessary to restore gauge invariance and other symmetries. The standard cancelled propagator argument fails because of kinematic restrictions imposed by D-instanton boundary conditions. As indicated in (18) the operator product of a $\text{NS} \otimes \text{NS}$ vertex operator coming close to the boundary produces a total derivative proportional to a ‘0/0’ expression because of the on-shell condition on the momentum of this vertex. The boundary contact term removes this term and restores the gauge invariance. This term is also necessary for spacetime supersymmetry and picture changing symmetry. The boundary term (11) effectively acts as a Lorentz rotation on spacetime spinors and modifies the action of the closed string supersymmetries on the open string fields. This fact might have some interesting consequences when the instanton induced effective vertices given by integration over the fermionic collective coordinates of a D-instanton [13] are considered in a non-trivial closed string background. It would be interesting to investigate whether there is a connection with recent suggestions of the appearance of central terms in the supersymmetry algebra associated with p-branes in the worldsheet R-NS formalism using spacetime supersymmetry charges in different pictures [18] or the incorporation of extra dimensions using $\text{R} \otimes \text{R}$ matter ghost fields [19].

Note that for D-brane boundary conditions with $p > -1$ the conventional cancelled propagator arguments apply and as stressed in [2] give the same result as the inclusion

of contact terms. The boundary conditions on the open string for a D p-brane with worldvolume in the $0, 1, \dots, p$ direction is given by

$$\left(\partial X^\mu + M_\nu^\mu \bar{\partial} X^\nu\right)|_{Im(z)=0} = 0, \quad \left(\psi^\mu + M_\nu^\mu \bar{\psi}^\nu\right)|_{Im(z)=0} = 0 \quad (62)$$

Where $M_\nu^\mu = \text{diag}(-\mathbf{1}_{p+1}, \mathbf{1}_{9-p})$. The OPE of a closed string AST vertex operator which approaches the D-p-brane boundary is now given by

$$\begin{aligned} \lim_{y \rightarrow 0} V(\zeta_{\mu\nu}^{AST}) &\sim \zeta_{\mu\nu}^{AST} M_\rho^\nu \frac{1}{(y)^{k.M.k+1}} k.M.k \psi^\mu \psi^\rho(x) + o(y^0) \\ &= \left(\frac{\partial}{\partial y} y^{k.M.k}\right) \zeta_{\mu\nu}^{AST} M_\rho^\nu \psi^\mu \psi^\rho(x) + o(y^0) \end{aligned} \quad (63)$$

And since $k.M.k = k_0^2 - k_1^2 + \dots k_p^2 - k_{p+1}^2 - \dots k_9^2$ it is possible if $p > -1$ to choose a physical on shell momentum k^μ such that $k.M.k > 0$ and the boundary contact term vanishes. Hence the contact terms are important for D-branes only if the closed string momentum k^μ is forced to vanish, which might be important when wrapped euclidian branes are considered.

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